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Hillslope subsurface flow similarity: Real-world tests of the hillslope Péclet number

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[1] Similarity analysis offers the ability to model hydrological response using quantifiable landscape descriptors. It is possible to develop similarity indices based on analytical solutions to the governing dynamic equations (Brutsaert, 2005). Berne et al. (2005) provide derivation of such a similarity index (the hillslope Péclet number) of subsurface flow and saturation for hillslopes with exponential width functions. They showed that the hillslope Péclet number depends only on geometric properties of the hillslope. Their work was validated using laboratory experiments conducted on constructed hillslopes with homogeneous soil structure and varying bedrock slope angle. This study applies the similarity analysis of Berne et al. (2005) to two data sets: (1) the trench hillslope study at the Maimai research catchment conducted by Woods and Rowe (1996) and (2) the isolated hillslope study near Troy, Idaho, United States, conducted by Brooks et al. (2004). The Maimai trench study was selected because it provides subsurface flow data from hillslopes with different planform geometries. The Troy hillslope study was selected because the experimental results of Brooks et al. (2004) provide an estimate of hydraulic conductivity consistent with the support scale of the hillslope. We estimated the hillslope Péclet number of the hillslopes on the basis of elevation data and reported values of average soil depth. This hillslope Péclet number quantifies the geomorphological control on how water moves through these hillslopes and creates a basis for comparison independent of hydraulic properties. We then estimated the first and second moments of the characteristic subsurface response function of each hillslope on the basis of subsurface flow data. To compare the empirical and theoretical moments, the hydraulic properties (saturated hydraulic conductivity and drainable porosity) of the hillslopes were related using a base flow recession analysis. Then we were able to derive the dimensionless moments of the hillslopes' observed characteristic response function using hydraulic conductivities reported in the literature. The agreement between the observed and theoretical moments shows the promise of implementing the hillslope Péclet number as a similarity parameter to describe first-order hydrological response in humid environments.

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1. Introduction

[2] Water interacts with the landscape as it turns from rain to stream flow. It not only flows downhill in response to topography, but also flows through the soil. This means that both geomorphology (hillslope shape) and pedology (soil properties) influence hydrological response. Similarity analysis tries to characterize this influence to allow for scaling and translation of hydrological response from one location to another. For example, topographic similarity indices [e.g., *Beven and Kirkby*, 1979; *O'Loughlin*, 1986] have been used to relate landscape positions to hydrological response. The appeal of such approaches is the ability to model hydrological response using quantifiable landscape descriptors. A main criticism of similarity approaches to date, however, has been the lack of analytical relationships between physical flow processes and these landscape descriptors [*Aryal et al.*, 2002].

[3] *Troch et al.* [2004], on the basis of an analytical solution of the linearized hillslope storage Bousinessq (hsB) equation, showed that a single dimensionless number, expressible in terms of quantifiable geometric properties (e.g., length, slope, soil depth, hillslope convergence rate), defines a hillslope Péclet number (Pe) for advective-diffusive subsurface flow dynamics along complex hillslopes. This follows from previous work in developing low-dimensional hillslope storage dynamics models capable of handling 3D hillslope structures in a simple way [e.g., *Troch et al.*, 2003]. Analytical solutions of this type may form the basis of hillslope similarity analysis [*Brutsaert*, 1994]. *Berne et al.* [2005] demonstrated this relating the hillslope Pe and the moments of characteristic response functions to quantify hillslope hydrological processes. This

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analytical relationship was confirmed with experimental results from a manufactured hillslope in various configurations. These authors stress the fact that the validity of their results is restricted to (1) the validity domain of the hsB equation (shallow soil mantle, streamlines parallel to the impervious bedrock, negligible influence of the unsaturated zone and absence of overland flow), (2) the validity domain of the linearization (constant slope angle, uniform hydraulic parameters and storage profile close to the mean profile), and (3) the considered initial and boundary conditions (steady state or uniform storage profile at the onset of drainage, zero head near outlet and no-flow boundary at divides).

[4] This research extends the hillslope work of *Troch et al.* [2003, 2004] and the similarity analysis of *Berne et al.* [2005]. The goal of this research is to test the ability of the similarity approach of *Berne et al.* [2005] to represent real-world hillslopes. This is done using topographic, pedologic, and subsurface flow data sets collected at two different field sites. The first was collected by *Woods and Rowe* [1996] at the Maimai research catchment located in New Zealand. The Maimai research catchment has a long history in hydrological field and model development research [*McGlynn et al.*, 2002]. Maimai was selected for this study for a number of reasons. First of all, the hydrology of the Maimai is compatible with the main assumptions underlying the hillslope storage Boussinesq model. *Vaché and McDonnell* [2006] state that the hillslopes at Maimai have highly transmissive soils underlain by largely impermeable bedrock [see also *McKie*, 1978], and *Mosley* [1979] observes that soils rarely fall below 90% of saturation. It can thus safely be assumed that lateral subsurface flow is the main runoff generation process, and that during most of the time storage in the unsaturated zone has only a minor delaying impact on the hillslope's hydrologic response. The latter assumption is also supported by the very small residence times reported for these hillslopes [*Vaché and McDonnell*, 2006]. Second, the trench study of *Woods and Rowe* [1996] provides a unique data set of subsurface flow dynamics in hillslopes with different planform geometries and compatible with the boundary conditions used by *Berne et al.* [2005] but with similar soils, vegetation, climate and geology. The second data set comes from the hillslope plot presented by *Brooks et al.* [2004] near Troy, Idaho, United States (from here forward, we refer to this site as the Troy hillslope). Again, the hydrology at this site is compatible with main assumptions underlying the hillslope storage Boussinesq model. Soils at the site are well drained and maintain a perched water table above a highly impermeable fragipan during wet (snowmelt) seasons [*Brooks et al.*, 2004]. In addition, interceptor drains were installed above and below the Troy hillslope site to reduce saturation excess runoff making lateral subsurface flow the main runoff generation process.

[5] Data sets such as those of *Woods and Rowe* [1996] and *Brooks et al.* [2004] are ideal for testing different hillslope similarity indices, including the one proposed by *Troch et al.* [2004] and *Berne et al.* [2005]. Moreover, by trenching the hillslopes at Maimai and installing interceptor drains below the plot at Troy, the investigators created downslope boundary conditions which are in line with the one assumed by *Troch et al.* [2004] and *Berne et al.* [2005] (near zero head at outlet) to derive their similarity index. The Maimai has a wealth of data available from previous field experiments and modeling investigations, which provides baseline information about hydraulic parameters needed to validate similarity indices such as the hillslope Pe number. The Troy site is unique in that hydraulic parameters (namely, hydraulic conductivity) have been measured at the appropriate (hillslope) support scale. With the wealth of data at both sites, hillslope Pe values can be estimated with a minimum number of assumptions. The purpose of this research is to explore the ability of the hillslope Pe to be used as a similarity parameter to represent real hillslopes. As more data sets come available, the authors hope that the hillslope Pe method of similarity analysis will prove to be a valuable analytical tool in characterizing hillslope processes in humid environments.

2. Defining the Hillslope Péclet Number

[6] The hillslope Pe number can be interpreted as the ratio of the characteristic diffusive timescale (τ_K) and the characteristic advective timescale (τ_U), defined for the middle of a hillslope. The characteristic advective time is defined as [*Berne et al.*, 2005]:

$$\tau_U = \frac{Lf}{2k(\sin\alpha - a_c pD\cos\alpha)} \tag{1}$$

and the characteristic diffusive time is [*Berne et al.*, 2005]:

$$\tau_K = \left(\frac{Lf}{2k}\right)\left(\frac{L}{2pD}\right)\left(\frac{1}{\cos\alpha}\right) \tag{2}$$

where f is the drainable porosity [−], L is the hillslope length [L], k is the hydraulic conductivity [L/T], α is the hillslope angle, pD is the average saturated depth [L], D is soil depth [L], p is a linearization parameter which ranges between 0 and 1, and a_c is the hillslope convergence rate [1/L]. The dimensionless ratio of τ_K and τ_U defines the hydrological similarity between hillslopes with respect to their characteristics response function (CRF) and, in turn, the hillslope Pe number:

$$\mathrm{Pe} = \left(\frac{L}{2pD}\right)\tan\alpha - \left(\frac{a_c L}{2}\right) \tag{3}$$

Brutsaert [1994, 2005] has proposed a similar dimensionless number for straight ($a_c = 0$) hillslopes, and calls it the Hillslope number (Hi). The Hi and Pe numbers define the dynamic response of straight hillslopes and hillslopes with exponentially converging/diverging width functions, respectively. This dynamic response was termed the characteristic response function (CRF) and is defined as the free drainage hydrograph, normalized by the total outflow volume, following an initial steady state storage profile corresponding to a constant recharge rate (initial condition 2 of *Berne et al.* [2005]). Since there is an assumption of an initial steady state storage profile, there is only one CRF for a given hillslope (in other words, there is only one way for a hillslope to recess from a steady state storage profile). *Berne et al.* [2005] provide a complete derivation of the analytical relationships between

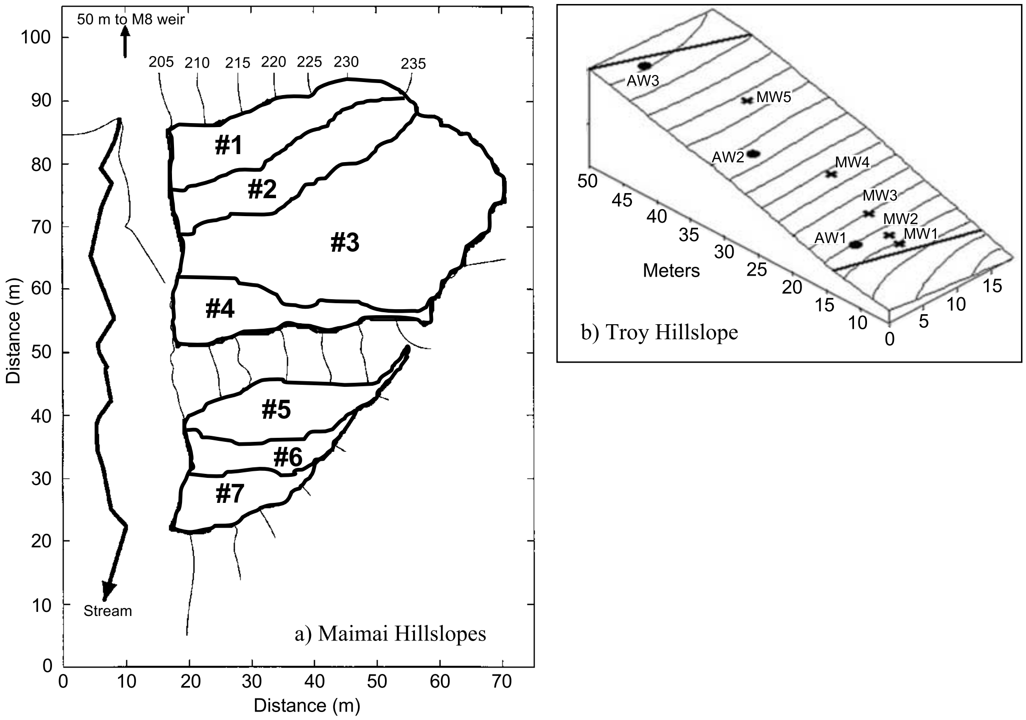

Figure 1. Topography and hillslope group identification for (a) the M8 catchment located in the Maimai research catchment [modified from *Woods et al.*, 1997] and (b) the Troy field site [from *Brooks et al.*, 2004].

the hillslope Pe number and the moments of this CRF. Because of the length of the analytical expressions, it is not feasible to reproduce them in this text. For those interested, scripts for implementing the analytical expressions are available from the authors.

3. Site and Data Descriptions

3.1. Maimai Hillslopes

[7] The Maimai research catchment, located on the west coast of the South Island of New Zealand, has been the host to numerous hydrological studies of the past 30 years (see *McGlynn et al.* [2002] for a thorough description of the various subcatchments and studies). This study focuses on the region referred to as catchment M8 and, more specifically, the trench excavated by *Woods and Rowe* [1996]. This trench was used to collect subsurface flow from the left bank of the mainstream draining catchment M8 (Figure 1a). This region of the Maimai is typified by short (<300 m) and steep slopes with a local relief of about 100 m [*Woods and Rowe*, 1996]. Shallow (average depth of 0.60 m) soils dominate the landscape. Because of the humid environment and topographic and soil characteristics, soils routinely stay within 10% of saturation by volume during the majority of the hydrologic year [*Mosley*, 1979]. Water draining from the M8 hillslope was collected at the interface of the highly permeable mineral soil and low-permeable subsurface by 30 sections of 1.7 m length PVC-lined subsurface flow troughs. Each trough was routed to a volume-calibrated tipping-bucket recording the total amount of subsurface flow. The full length of the trench, however, was not continuously instrumented along the M8 hillslope. Because of excessively deep soils and concerns for slope stability, the section of trench between troughs 20 and 21 was not excavated down to the low-permeable subsurface creating disconnect in the instrumentation along the trench face. Data was collected in the trench from 24 January 1993 through 15 May 1993. During this period, four major runoff-producing events were observed and recorded. These events are discussed in detail by *Woods and Rowe* [1996].

3.2. Troy Hillslope

[8] The Troy hillslope site is located 8 km north of Troy, Idaho, United States (Figure 1b). The site encompasses one hillslope isolated from the surrounding landscape using tile lines and plastic and metal sheeting. The tile line downslope of the hillslope was installed to collect and divert lateral flow to an automated tipping bucket recording at 15 min intervals. The upslope tile line was installed to divert incoming lateral subsurface flow. The whole region is underlain by a relatively impervious fragipan layer at a depth of about 0.65 m. This condition results in the development of a perched water table during winter to spring (wet) conditions. Soil at the site is primarily a silt loam classified as a Luvisol using the World Reference Base for Soil Resources and provides well defined A, B, and E horizons. The site receives mean annual precipitation between 500 and 830 mm with more than 60% falling as low-intensity rain or snowmelt from November to April. Prior to the 1960s, the land was forested with pine and fir, but this was cleared for farming (grain/legume rotation) from the mid 60s through 1994 when it was seeded with perennial grasses. Data was collected (e.g., tile line outflow) at the site from 26 March 2002 through 14 April 2002. This covered a period of five days of snowmelt events (with the largest occurring on the last day of snow cover) followed by ten days of drainage and ending with four days of rainfall. A

Table 1. Geometric and Hydraulic Parameter Values Used for Calculating Hillslope Pe and the First and Second Moments of the Observed CRF for the Maimai Hillslopes and the Troy Hillslope

	Maimai Hillslopes							
Troughs	Hillslope 1 Troughs 1 to 6	Hillslope 2 Troughs 7 to 10	Hillslope 3 Troughs 11 to 14	Hillslope 4 Troughs 15 to 20	Hillslope 6 Troughs 22 to 25	Hillslope 7 Troughs 26 to 30	Troy Hillslope	Units
			Parameters					
Area A	246	292	927	209	114	132	630	m^2
Hillslope length L	30	43	50	30	17	15	35	m
Channel length L_c	10.2	6.8	6.8	10.2	6.8	8.5	18	m
Hillslope angle α	39.8	34.9	35.0	39.8	49.6	45.0	11.5	deg
Average soil depth D	0.60	0.60	0.60	0.60	0.60	0.60	0.65	m
Convergence rate a_c	−0.015	0	0.035	−0.027	0	0	0	m^{-1}
Hydraulic conductivity k	0.25	0.25	0.25	0.25	0.25	0.25	0.025	m/hr
Drainable porosity f	0.08	0.08	0.08	0.08	0.18	0.18	0.01	
Linearization parameter p	0.3	0.3	0.3	0.3	0.3	0.3	0.3	
Hillslope Péclet number Pe	69.7	83.3	96.3	69.9	55.6	41.7	18.2	
			Observed CRF					
First moment	3.8	6.2	7.7	2.8	5.2	3.9	23.2	hr
Second moment	25.9	67.0	84.8	12.9	46.6	29.4	544.5	hr^2
Characteristic diffusive time τ_K	491	945	1,279	491	439	392	621	hr
First CRF dimensionless moment $\times 10^{-2}$	0.77	0.66	0.60	0.57	1.18	0.99	3.74	
Second CRF dimensionless moment $\times 10^{-4}$	1.07	0.75	0.52	0.54	2.42	1.91	14.12	

comprehensive site description is provided by *Brooks et al.* [2004].

4. Applying Similarity Analysis

[9] Extensive field and modeling studies have been performed in the Maimai catchment [e.g., *Woods and Rowe*, 1996; *McDonnell*, 1990; *McGlynn et al.*, 2004; *Vaché and McDonnell*, 2006]. For the Troy site, we rely on data collected and presented in the work of *Brooks et al.* [2004]. While not as extensive as the work at Maimai, the intensive nature of this study make the data appropriate for this study. These studies provide the framework for determining the geometric parameters needed in (3) to compute hillslope Pe for use as a similarity parameter. The relationship between hillslope Pe and the theoretical CRF moments [*Berne et al.*, 2005] gives a dimensionless descriptor of the hydrological response of a hillslope based solely on the geometry. To compare this with the actual "observed" hydrological response, we need to define a representative timescale in terms of the characteristic diffusive time (τ_K) based on hydraulic parameters (i.e., hydraulic conductivity and drainable porosity). On the basis of the observed hydrological response at each site (i.e., the recession from an approximate steady state), we can define an observed CRF and make it dimensionless with τ_K. The moments of that dimensionless CRF allow for comparison between observed and the theoretical (geometry-based) dimensionless CRF moments. The following sections outline this procedure in detail for the Maimai hillslopes and the Troy hillslope.

4.1. Evaluating Hillslope Pe

4.1.1. Maimai Geometry

[10] The M8 hillslope was divided into 7 subhillslopes (Figure 1a) on the basis of topography by *Woods and Rowe* [1996]. From these subhillslopes, 6 subhillslopes (hereafter referred to collectively as the Maimai hillslopes and individually by number) were used to determine geometric parameters and hillslope Pe (Table 1). Hillslope 5 (draining through trough 21) showed no hydrological response to rainfall and possibly drained into the unsampled (unexcavated) region of the hillslope. Thus it was omitted from this study. Hillslope area (A), length (L), and angle (α) were determined directly from the elevation surveys (R. Woods, personal communication, 2006). Channel length (L_c) was taken as the number of trenches draining an individual hillslope multiplied by the unit trench length (1.7 m). Soil depths at Maimai ranged from 0.15 m to 0.30 m on ridge tops and 0.60 m to 2.0 m along the midslopes and central hollow [*Freer et al.*, 1997]. Average soil depths of 0.60 m have been reported throughout the literature [*McDonnell*, 1990; *Woods and Rowe*, 1996]. We used this average value as an effective value to calculate hillslope Pe values for the Maimai hillslopes. In (3), the term pD estimates the average saturated depth of the hillslope reflecting the time-space average of the saturated storage during the recession from an initial steady state storage profile, corresponding to a constant recharge rate. The initial condition for the Maimai hillslopes in this analysis was assumed on the basis of this steady state storage profile (corresponding with IC 2 given by *Berne et al.* [2005]).

[11] The hillslope convergence rate (a_c) for each individual Maimai hillslope was determined assuming exponential ($a_c \neq 0$) or uniform ($a_c = 0$) width functions. While exponential and uniform width functions may not be appropriate for every hillslope in the world, they provide a simple, easily defined width function capable of representing the shape of many hillslopes and are assumed in this analysis. To check the assumption of exponential width functions, we fitted equations to observations of width as a function of distance upslope from the trough for hillslopes 1, 3, and 4 (the remaining hillslopes were assumed to be uniform). This resulted in exponential width functions for hillslope 1, 3, and 4 with fits of R^2 = 0.78, 0.96, and 0.95, respectively. Thus exponential width functions are adequate descriptors for the hillslope widths at Maimai. From a water

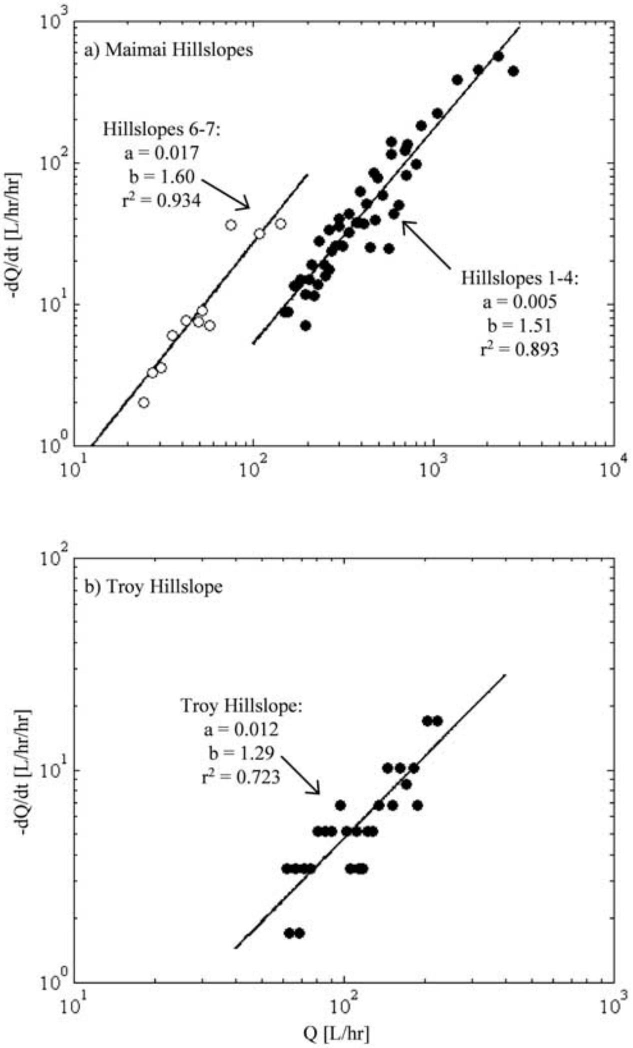

Figure 2. Brutsaert-Nieber recession flow analysis for (a) Maimai hillslopes 1–4 (open circles) and 6–7 (closed circles) and (b) Troy hillslope. Lines correspond to the relation given in equation (4).

balance perspective, however, there is a more appropriate manner to define the exponential width function. When using an exponential width function, outlet width was set as the channel length (L_c) and a_c was adjusted while preserving its relation to hillslope area, $A = L_c(\exp(a_c L) - 1)/a_c$ [*Troch et al.*, 2003; *Berne et al.*, 2005]. In this manner, a_c was determined for hillslopes 1, 3, and 4. Finally, the linearization parameter p, which is nonphysical, can only be determined through calibration [*Berne et al.*, 2005]. In this study we avoid calibration by selecting a theoretical value of 0.30 that can be derived from exact solutions to the Boussinesq equation [*Brutsaert*, 1994]. However, this value is applicable only for small to intermediate times after the onset of free drainage, and is therefore expected to affect the higher-order moments of the CRF. For this reason, in this study we will only investigate the relationship between the first two (central) moments of the CRF and the hillslope Pe. On the basis of these geometric parameters, a hillslope Pe number was defined for each of the individual Maimai hillslopes using (3) (see Table 1).

4.1.2. Troy Geometry

[12] The Troy site has constructed boundaries making geometric parameters easily defined. Hillslope length (L), Channel length (L_c), and angle (α) were determined directly from the elevation surveys. Hillslope area (A) was simply calculated as $L \times L_c$. An effective soil depth of 0.65m was defined on the basis of the average depth to fragipan for the hillslope. Again, the initial condition for the Troy hillslope in this analysis was assumed on the basis of a steady state storage profile (corresponding with IC 2 given by *Berne et al.* [2005]). Hillslope convergence rate (a_c) for the Troy hillslope was taken as 0 corresponding to a uniform width function. Similar to the Maimai site, the linearization parameter p was set to 0.3 to avoid calibration. On the basis of these parameters, the hillslope Pe number was defined for the Troy hillslope using (3) (see Table 1).

4.2. Evaluating Hydraulic Parameters

[13] Numerous and highly variable values for hydraulic conductivity have been reported in the literature for both the Maimai's M8 catchment and the Troy site (see *McGlynn et al.* [2002] and *Brooks et al.* [2004], respectively, for discussion). Obviously, there is much variation and uncertainty associated with measuring hydraulic conductivity at the hillslope scale. This uncertainty translates directly into uncertainty about drainable porosity (f). To circumvent this, we use recession analysis to reduce the number of degrees of freedom in defining hydraulic parameters by linking f directly to k.

[14] For this study, we use reported field measurements for hydraulic conductivity in conjunction with the Brutsaert-Nieber recession flow analysis to define an implicit value of drainable porosity as a function of hydraulic conductivity. The recession flow analysis technique of Brutsaert-Nieber recession flow analysis technique [*Brutsaert and Nieber*, 1977] describes groundwater outflow from an unconfined aquifer (Q) in time (t) with

$$\frac{dQ}{dt} = -aQ^b \tag{4}$$

where a and b are constants. Consistent with the Brutsaert-Nieber technique, a log-log plot was created of $-dQ/dt$ against Q to determine the constants a and b for both Maimai and Troy (Figures 2a and 2b, respectively). Note that for this analysis in the Maimai, the period of record corresponding to the storm event used in estimating the empirical CRF moments (see Section 4.3) was eliminated to avoid its influence on the recession characteristics; however, because of the limited record of observations, at Troy this was not an option. Also, the Maimai hillslopes were grouped into two subcatchments (i.e., hillslopes 1–4 and 6–7) on the basis of the observed physical disconnect (Figure 1a) and observations from previous researchers (B. McGlynn, personal communication). Thus separate values of a and b were obtained for each hillslope grouping; however, both groups were characterized (as was the Troy site) by long-time outflow rate solutions to the Boussinesq equation (i.e., $b \approx 3/2$). A well defined expression for relating the intercept value (a) to hydraulic conductivity (k) and drainable porosity (f) exists from *Brutsaert* [2005]:

$$a = \frac{4.8038 k^{1/2} L_c}{f A^{3/2}} \tag{5}$$

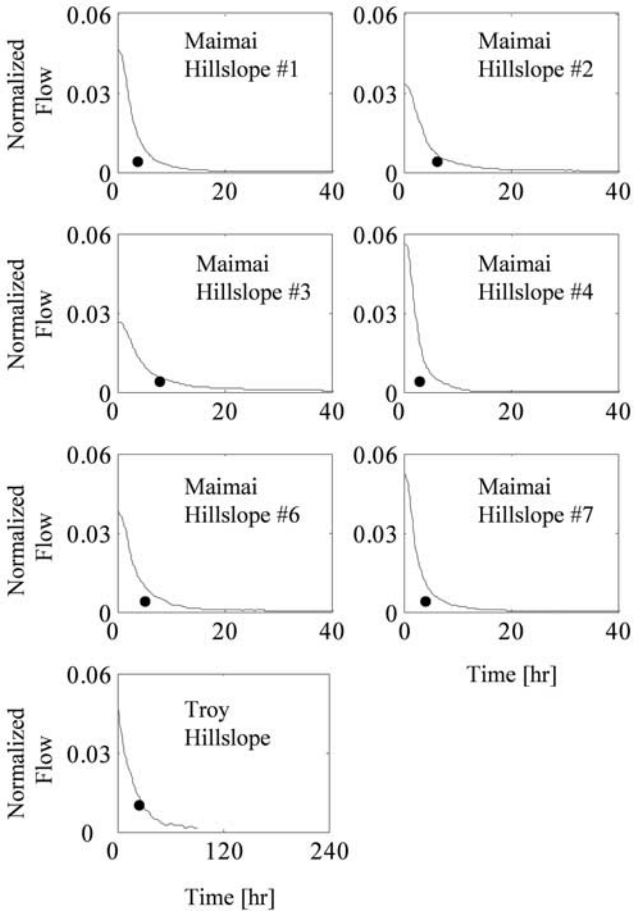

Figure 3. Observed CRF for each of the Maimai hillslopes based on the 26 January 1993 event from *Woods and Rowe* [1996] and for the Troy hillslope (note the change in the time axis) based on the 30 March 2002 event from *Brooks et al.* [2004]. The point on each curve shows the location of the first CRF moment.

where A is the drainage are of the catchment [L^2] and L_c is the length of the channel draining the catchment [L]. For long recession times, (5) provides an exact solution to the Boussinesq equation [*Brutsaert and Nieber*, 1977; *Troch et al.*, 1993]. At Maimai, A was calculated from the sum of individual hillslope areas and L_c was taken as the number of trenches draining each subcatchment multiplied by the unit trench length (1.7 m). At Troy, A was calculated by multiplying the plot length by the plot width and L_c was taken as the plot width. It should be noted that, as a first attempt, the Brutsaert-Nieber technique was performed treating the entire M8 hillslope as a single catchment. This resulted in a relationship similar to that seen for the hillslopes 1–4, but which failed to effectively represent the hydraulic parameters of hillslopes 6–7. While (5) was developed for horizontal hillslopes, *Zecharias and Brutsaert* [1988] found the relationship remains unchanged for increasing slope of the bedrock; therefore we can safely apply it to the Maimai and Troy hillslopes. Rearranging (5) gives an implicit value of drainable porosity as a function of observed hydraulic conductivities.

[15] In general, observed values of k are highly variable because of the difficulty in identifying the appropriate support scale for k observations and because of the heterogeneity of measurements in space. For this study, we assume that observations of k approximate an effective k that is capable of representing subsurface flow dynamics at the hillslope scale. This is required to evaluate the first and second dimensionless moments from the observed CRFs, however, not required for calculation of the hillslope Pe number. It should be noted that heterogeneity in k influences the actual values for the first and second moments of the CRF and that assuming an effective k will, at best, provide an approximation.

[16] For Troy, the work of *Brooks et al.* [2004] directly determined k at the hillslope scale for each soil horizon (A, B, and E). For this study we adopt the values for the E horizon as the most representative when considering how hillslope outflow is collected (i.e., using a tile line buried at the interface of soil and fragipan). The E horizon provides the lowest observed k value from *Brooks et al.* [2004] and is the limiting factor for subsurface flow. Using this value of k, a drainable porosity of 0.01 was calculated for the Troy hillslope (Table 1). This low calculated drainable porosity could be due to the extremely saturated state at the beginning of the recession and would be similar to observations by *Hilberts et al.* [2005] for finer grained soils.

[17] For the Maimai, numerous and highly variable values for hydraulic conductivity have been reported in the literature for the Maimai's M8 catchment. *McDonnell* [1990] reported values ranging from $\ll$0.005 m/hr in poorly drained hollows to 0.25 m/hr on well-drained nose slopes on the basis of field measurements. *Webster* [1977] gave average hydraulic conductivities of 0.25 m/hr in the mineral soil on the basis of laboratory conducted core measurements. *Woods et al.* [1997] were able to model calibrate values for hydraulic conductivities relative to runoff data for troughs 26 through 30 of 0.25 m/hr (although they observed similar results when assuming k values as low as 0.18 m/hr). In a review of work at all the Maimai catchments, *McGlynn et al.* [2002] provide a range of hydraulic conductivities from 0.01 to 0.30 m/hr with *McGlynn and McDonnell* [2003] giving an average of 0.25 m/hr. For this study, this average of value of k of 0.25 m/hr is assumed to be the effective k for the hillslopes. Since this value is based at best on limited observations or model calibrations, it intrinsically contains uncertainty, but we adopt it as an effective parameter for wont of a more appropriate value. Using this value of k, drainable porosities of 0.08 and 0.18 were calculated for the Maimai subcatchments consisting of hillslopes 1–4 and 6–7, respectively (Table 1). The difference between the subcatchments reflects the difference observed in the recession analysis (Figure 2a). By defining the effective k and, subsequently, determining f for each site, we can translate the observed hydrological responses (i.e., observed CRF) into dimensionless space and thus compare them to the theoretical hydrological response described with the hillslope Pe number (i.e., the basis of our similarity analysis).

4.3. Observed CRF and Similarity Analysis

[18] The observed first and second "real-time" CRF moments (Table 1) were calculated for each of the Maimai hillslopes and the Troy hillslope using normalized recession hydrographs (Figure 3) similar to the method outlined by *Berne et al.* [2005]. The first moment of the observed CRF is a measure of the traveltime of the mean hillslope response.

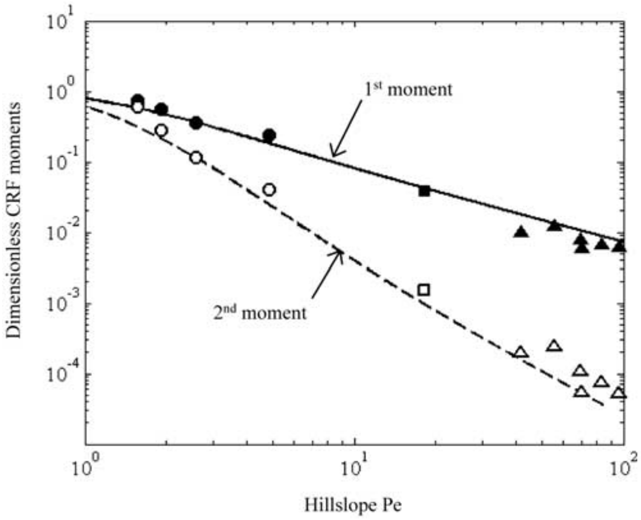

Figure 4. Observed CRF related to the hillslope Pe for Maimai hillslopes (solid triangles for first moment and open triangles for second moment) and Troy hillslope (solid squares for first moment and open squares for second moment). Circles (solid for first moment and open for second moment) show the empirical results reported by *Berne et al.* [2005]. The solid curve shows the relation from *Berne et al.* [2005] between theoretical CRF and hillslope Pe for the first moment, and the dashed curve shows it for the second moment.

From the normalized recession hydrographs, it is seen that at Maimai the longest response time comes from hillslope 3 (7.7 hours) and the shortest comes from hillslope 4 (2.8 hours) that correspond to the most convergent and most divergent hillslopes, respectively. This is similar to the results reported by *Troch et al.* [2003]. The CRF for each of the Maimai hillslopes was defined from the outflow from a 10 hour rain event of 53mm occurring on 26 January 1993 [see *Woods et al.*, 1997, Figure 3a]. The prestorm hillslope wetness was typical for the Maimai region (API_7 = 20 mm where API_7 is a 7-day antecedent precipitation index defined by $\sum_{i=1}^{7} p_i/i$ where p_i is the daily rainfall on the ith day before the storm). This API_7 is sufficiently large to provide a wet initial condition for the Maimai hillslopes such that unsaturated storage has a minimal delaying effect on subsurface outflow. Moreover, this wet initial condition coupled with a 53mm rainfall event creates a scenario where the full flow of the storm event approximates steady response [*Woods et al.*, 1997]. For the Troy site, the CRF is defined using the recession event following the onset of snowmelt (30 March 2002). Prior to this period, the observed water table is only 10 cm below the hillslope soil surface [*Brooks et al.*, 2004] providing sufficiently wet initial conditions. Therefore the resulting recessions for both sites can be used to estimate the dimensionless moments of the individual hillslopes for comparison with the theoretical first and second CRF moments (Figure 4 and Table 1).

5. Discussion

[19] In this study we have applied a theoretically derived similarity index (the hillslope Péclet number) for shallow subsurface flow along hillslopes to the real-world data of *Woods and Rowe* [1996] and of *Brooks et al.* [2004]. The hillslope Pe number predicts subsurface flow response dynamics when a number of simplifying assumptions apply: (1) the hillslopes are characterized by shallow soils underlain by impermeable bedrock, streamlines are essentially parallel to the impervious bedrock, storage in the unsaturated zone has a negligible delaying effect on subsurface flow response and the absence of overland flow, (2) constant bedrock slope angle, uniform hydraulic parameters and storage profile close to the mean profile, and (3) zero head boundary condition near outlet and no-flow boundary at divides. It is impossible to find real hillslopes to meet all of the above requirements. Nevertheless, those used in this study provide a unique opportunity to test the validity and limitations of the proposed similarity index. All Maimai hillslopes have shallow highly transmissive soils overlying bedrock that is essentially impermeable and subsurface flow as the dominant runoff generating process. Similar conditions exist at Troy via well-drained soils, fragipan, and interceptor drains. Unsaturated zone storage is not important in defining the characteristic response time under wet conditions, and such conditions apply for most of the time at Maimai and during snowmelt at Troy. While the slope angles are steep at Maimai and more subdued at Troy, both can be approximated as constant. Both experimental setups to collect subsurface flow were such that zero-head boundary conditions are good first-order approximations.

[20] Large uncertainties remain for the other simplifying assumptions: uniform hydraulic parameters, storage profile during drainage close to mean profile (the linearization assumption) and no-flow boundary conditions at the divides (although this is minimized at Troy because of an interceptor drain). We chose to use a base flow recession analysis to estimate relationships between the hydraulic parameters (k and f) to reduce the number of free parameters. While this can be seen as “circular” in that we use recession analysis to link hydraulic parameters used to make a CRF, which is a hydrograph recession, dimensionless, it allows us to avoid reliance on direct measures of drainable porosity. Such measurements are difficult to make in a way that they meaningfully capture the water table dynamics of a hillslope. Even though we get around measurements of drainable porosity, we still require measurements of hydraulic conductivity. There is uncertainty associated with measures of k at the hillslope scale, but adopting effective values (i.e., average for Maimai and hillslope support scale for Troy) it is possible to approximate the first and second CRF moments. The linearization assumption relates to the estimation of an appropriate value for p. The parameter p is nonphysical and scales the available storage capacity to a space-time average storage. We avoided fine tuning of our results by fixing its value to the theoretical value of 0.3. More research is currently focusing on the relationship between the average saturated storage and climate, soil and geometry characteristics of the hillslopes. Finally, uncertainties remain concerning the hillslope divides derived from surface elevation information at Maimai. To reduce this uncertainty, the method proposed by *Woods and Rowe* [1996] (namely, to group individual troughs into larger subhillslope units) was employed.

[21] Our results demonstrate that the hillslope Pe number has the potential to describe, as a first-order approximation, hillslope subsurface response on the basis of geometric characteristics. The difficulty in estimating appropriate hillslope hydraulic parameters is not only related to the analyzed similarity index, but results from the ubiquitous heterogeneous nature of the subsurface. The latter represents one of the most prominent problems in hydrology. Since the hydraulic conductivities used in this study approximate the theoretical curve (in terms of magnitude and shape among and between sites), it can be safely assumed that an effective hydraulic conductivity for each hillslope exists that would allow the points to fall perfectly on the curve. When differences in hydraulic parameters between hillslopes can be minimized (as in Maimai), we have shown that the hillslope Pe number capture the essence of the differences in the response between the individual hillslopes. This is demonstrated in the similarity in slope of the points from the Maimai hillslopes and the theoretical curve. To further demonstrate this, we have plotted the experimental results from *Hilberts et al.* [2005] and *Berne et al.* [2005] in Figure 4. For these points (circles on Figure 4), a laboratory hillslope was manufactured and tested under drainage conditions for various convergence rates and slopes. Since the hillslope was manufactured, exact values of hydraulic and geometric parameters could be determined and measurement error decreased. These points, not surprisingly, agree extremely well with the theoretical curve. Note that the hillslope Pe values evaluated for the laboratory hillslope configurations were much lower than those evaluated for the Maimai hillslopes and similar to that at Troy. This is because Maimai hillslopes are longer and have much steeper bedrock slope angles than those used in the laboratory experiments of *Hilberts et al.* [2005] (i.e., maximum slope of 15% instead of the >30% in Maimai) and the Troy hillslope. Since advection is the dominant flow process in such situations [*Beven*, 1982], advection timescales are much smaller than the diffusive timescales causing the larger hillslope Pe values. This type of quantified comparison between different hillslopes is what makes the hillslope Pe a powerful similarity parameter.

[22] The hillslope Pe number provides a way to isolate the relative geomorphologic controls of hillslopes on the travel of water through a catchment. Regardless of how this is done, the strength of the hillslope Pe number as a similarity parameter lies in its ability to discern hillslope responses in a given hydrological setting solely on the basis of geometry. Difficulty arises when real-time mean response time is needed for a hillslope. It is then necessary to either estimate or directly measure hydraulic parameters at the support scale for a hillslope. Direct measurements are difficult (but not impossible) to make because of inherent heterogeneity and the limited ability to physically define what hydraulic parameters mean at the support scale of a hillslope. For example, the two Maimai subcatchments (i.e., hillslopes 1–4 and 6–7), based on the Brutsaert-Nieber analysis, have different hydrological recession characteristics and thus different estimated drainable porosities assuming the same hydraulic conductivities (Table 1); however, the responses of the six hillslopes at Maimai can be quantified and compared relative to one another using the hillslope Pe number, without the need for quantifying the hydraulic parameters by simply estimating normalized and dimensionless response functions for each hillslope. Since this allows for a comparison that is independent of scale, it is also possible to place these hillslopes in the context of the Troy and laboratory hillslope configurations. Thus, independent of hydraulic parameters, we can begin to address how we expect water to move through these hillslopes without observations (i.e., the problem of ungauged basins).

6. Concluding Remarks

[23] This study provides the next step in testing the ability of the hillslope Pe as a similarity parameter capable of capturing complex hillslope geometries. The values of hillslope Pe calculated for the Maimai and Troy hillslopes showed good agreement with the theoretical relationship between hillslope Pe and the first and second dimensionless CRF moments. This supports previous experimental results from *Hilberts et al.* [2005] for a manufactured hillslope. To further support the use of the hillslope Pe as a similarity parameter, the theoretical curve needs to be populated with data from other hillslope-scale field studies in different climatic regions. This will demonstrate how spatial heterogeneity in real-world parameters influences the hillslope Pe and offer insight on how to upscale to the catchment scale.

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