A low-dimensional physically based model of hydrologic control of shallow landsliding on complex hillslopes

Ali Talebi, Remko Uijlenhoet and Peter A. Troch

Hillslopes have complex three-dimensional shapes that are characterized by their plan shape, profile curvature of surface and bedrock, and soil depth. To investigate the stability of complex hillslopes (with different slope curvatures and plan shapes), we combine the hillslope-storage Boussinesq (HSB) model with the infinite slope stability method. The HSB model is based on the continuity and Darcy equations expressed in terms of storage along the hillslope. Solutions of the HSB equation account explicitly for plan shape by introducing the hillslope width function and for profile curvature through the bedrock slope angle and the hillslope soil depth function. The presented model is composed of three parts: a topography model conceptualizing three-dimensional soil mantled landscapes, a dynamic hydrology model for shallow subsurface flow and water table depth (HSB model) and an infinite slope stability method based on the Mohr–Coulomb failure law. The resulting hillslope-storage Boussinesq stability model (HSB-SM) is able to simulate rain-induced shallow landsliding on hillslopes with non-constant bedrock slope and non-parallel plan shape. We apply the model to nine characteristic hillslope types with three different profile curvatures (concave, straight, convex) and three different plan shapes (convergent, parallel, divergent). In the presented model, the unsaturated storage has been calculated based on the unit head gradient assumption. To relax this assumption and to investigate the effect of neglecting the variations of unsaturated storage on the assessment of slope stability in the transient case, we also combine a coupled model of saturated and unsaturated storage and the infinite slope stability method. The results show that the variations of the unsaturated zone storage do not play a critical role in hillslope stability. Therefore, it can be concluded that the presented dynamic slope stability model (HSB-SM) can be used safely for slope stability analysis on complex hillslopes. Our results show that after a certain period of rainfall the convergent hillslopes with concave and straight profiles become unstable more quickly than others, whilst divergent convex hillslopes remain stable (even after intense rainfall). In addition, the relation between subsurface flow and hillslope stability has been investigated. Our analyses show that the minimum safety factor (FS) occurs when the rate of subsurface flow is a maximum. In fact, by increasing the subsurface flow, stability decreases for all hillslope shapes. Copyright © 2008 John Wiley & Sons, Ltd.

Keywords: hillslope stability; subsurface flow; HSB-SM

Introduction

Hillslopes can be considered as the basic landscape elements of many catchments. A proper understanding of the interaction and feedbacks between hillslope forms and the processes responsible for hillslope hydrology and stability are of great importance for catchment scale land management. Hillslope failures are complex natural phenomena that pose a serious natural hazard in many countries. Consequently, not only are considerable financial costs suffered, but also major ecological and environmental problems may arise in larger geographical areas (Sidle and Ochiai, 2006). To
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prevent or mitigate these damages, hillslope stability analysis requires an understanding and evaluation of the processes that are affected by the hydrologic behavior of the hillslopes.

The relationship between rainfall, water table fluctuations and landslide movement is often difficult to establish. Keefer and Larsen (2007) state that, although major causes of landslides are well known, predicting just where and when a landslide will occur continues to be a complex problem, because the properties of earth materials and slope conditions vary greatly over short distances, and the timing, location and intensity of triggering events are difficult to forecast. Shallow slope failures, in general, are controlled by surface topography through shallow subsurface flow and increased soil saturation (Montgomery and Dietrich, 1994; Iida, 1999; Borga et al., 2002). Many studies (e.g. Sidle and Swanston, 1982; Harp et al., 1990; Anderson and Sitar, 1995; Iverson, 2000; Dhakal and Sidle, 2004; Iida, 2004; Rosso et al., 2006) have indicated that hillslope instability can be caused by increased subsurface pore pressures during periods of intense rainfall, which reduce the shear strength of hillslope materials. Although in these studies topography has been reported as an important factor in slope stability, in most of the models applied only slope angle has been investigated. From a slope stability viewpoint, other topography parameters such as profile curvature and plan shape are sometimes equally important. Former (Beven and Kirkby, 1979; Sidle, 1984; Montgomery and Dietrich, 1994) and more recent studies (e.g. Tsuoyama et al., 2000; Troch et al., 2002, 2003; Hilberts et al., 2004; Berne et al., 2005; Rezzoug et al., 2005; Sidle and Ochiai, 2006) have shown that, in addition to bedrock slope, hillslope form as represented by plan shape and profile curvature is an important control on subsurface flow response.

Because of the strong relation between hydrological processes, hillslope shape and slope stability, efficient tools are needed to investigate the effect of complex topography on slope stability at the landscape scale. Existing tools either neglect topographic curvature or model it in a very complex computationally inefficient manner. The main purpose of this study is therefore to present a dynamic low dimensional but physically based model that includes both hydrological processes (saturated and unsaturated zone storage) and a stability model (based on the infinite slope assumption) for hillslopes with different topographic characteristics.

To study the effect of topography on rain-induced shallow landsliding, some researchers (e.g. Montgomery and Dietrich, 1994; Sidle and Wu, 1999; Dhakal and Sidle, 2003, 2004; Talebi et al., 2007) used the kinematic wave (KW) assumption for hillslope hydrology and showed that, in addition to bedrock slope, plan shape and slope curvature play important roles in hillslope stability in the case of a steady-state hydrology. However, by comparing the KW model with the fully three-dimensional Richards equation, Hilberts et al. (2004) showed that for convergent slope forms the KW model loses its ability to accurately describe water table dynamics and the resulting hillslope drainage. Since the dynamic response of hillslopes is strongly dependent on plan shape, slope curvature and slope angle (Troch et al., 2002; Hilberts et al., 2004), a three-dimensional model of dynamic hillslope hydrology would be necessary for stability analysis of complex hillslopes.

Recent studies (Troch et al., 2003; Paniconi et al., 2003; Hilberts et al., 2004, 2007) have generalized the Boussinesq equation to account for the three-dimensional soil mantle in which subsurface flow processes take place. This hillslope-storage Boussinesq (HSB) equation is formulated by expressing the continuity and Darcy equations in terms of soil water storage as the dependent variable. In this model, the method proposed by Fan and Bras (1998) is used to collapse the three-dimensional soil mantle of complex hillslopes into a one-dimensional drainable pore space. The resulting HSB model shows that the dynamic response of complex hillslopes during drainage and recharge events depends very much on the slope angle, plan shape and slope curvature. Because of the ability of the HSB model to analyze hydrological processes on complex hillslopes in a very (computationally) efficient way, it is a good candidate to combine with the infinite slope stability method. The resulting model (HSB-SM) can be used for dynamic slope stability analyses on complex hillslopes.

To keep the model low-dimensional, the average soil moisture in the unsaturated zone has been calculated according to Campbell’s method (Campbell, 1974), using Darcy’s law with the unit-gradient assumption. As a result, in the HSB-SM, the effect of the temporal variations of the unsaturated zone storage has been neglected. To investigate the effect of this simplification, a coupled model of the dynamic unsaturated and saturated zones is needed. Recently, Hilberts et al. (2007) developed a model that couples the one-dimensional Richards equation for vertical unsaturated flow and the HSB equation for lateral saturated flow along complex hillslopes. By introducing the unsaturated zone matric pressure head as a system state and reformulating the derived equations into state-space notation, they have solved the coupled system simultaneously as a set of ordinary differential equations. Considering the importance of the capillary fringe to groundwater flow, this component has also been included in the HSB flow domain (see Hilberts et al., 2007). Their model allows for an accurate investigation of the relationship between rainfall intensity, drainable porosity, unsaturated storage and recharge.

To investigate the importance of neglecting the variations of the unsaturated zone storage in the HSB-SM, the coupled HSB model (Hilberts et al., 2007) is combined with the infinite slope stability method. Moreover, the effect of unsaturated soil moisture on soil cohesion is incorporated in the slope stability model.
Hence, in this paper we present a methodology to investigate the effect of rainfall and water table variations on slope movement on complex hillslopes by coupling an integrated hydrologic model for the saturated and unsaturated zone with the infinite slope method for hillslope stability analysis. Specifically, the study aims (i) to present a dynamic hillslope stability model (applicable on complex hillslopes), (ii) to investigate the relation between rainfall, soil moisture storage, subsurface flow and hillslope stability and (iii) to investigate the changes of hillslope stability with respect to plan shape and slope curvature during rainfall.

Model Formulation

In this study we combine a dynamic hillslope hydrology model with the infinite slope stability assumption for hillslopes with different topographic characteristics. Hence, this model incorporates three important aspects of a hillslope: its topography, hydrology and stability.

Hillslope topography

To study the effect of topography (plan shape and profile curvature) on rain-induced shallow landsliding, Talebi et al. (2007), following Evans (1980), characterized hillslopes by the combined curvature in the gradient direction (profile curvature) and the direction perpendicular to the gradient (contour or plan curvature). The surface of an individual hillslope is represented by the following bivariate function (Evans, 1980):

\[ z(x', y) = E + H(x'/L')^n + \omega y^2 \]  

where \( z \) is the elevation, \( x' \) is the horizontal distance from the outlet, \( y \) is the horizontal distance from the slope centre in the direction perpendicular to the length direction (the width direction), \( E \) is the minimum elevation of the surface above an arbitrary datum, \( H \) is the maximum elevation difference defined by the surface, \( L' \) is the horizontal length of the hillslope, \( n \) is a profile curvature parameter and \( \omega \) is a plan curvature parameter.

The presented model (HSB-SM) is applied to nine distinct hillslope types, which can be viewed as a first-order approximation of the landscape elements (Troch et al., 2002) that constitute a catchment. Figure 1 shows a hillslope with a three-dimensional soil mantle on top of an impermeable layer and a straight bedrock profile, explaining the symbols \( w, L, D \) and \( \beta \) in this study. By using Equation (1) and allowing profile curvature (defined by \( n \)) to assume values less than, equal to or greater than unity and plan curvature (defined by \( \omega \)) to assume either positive, zero or negative values, one can define different basic geometric relief forms. Figure 2 illustrates nine basic hillslope types that are formed by combining three plan and three profile curvatures. These nine hillslopes represent a wide range of hillslope types traditionally considered in hydrology and geomorphology (see also Tsukamoto and Ohta, 1988). The parameters for Equation (1) are different for each of these nine hillslopes, and are listed in Table I. The values of the hydrological and geotechnical parameters have been listed in Table II. The horizontal length of the nine hillslopes (measured along the bedrock) is chosen to be constant \( (L = 100 \, \text{m}) \). For different hillslopes within a catchment each individual hillslope type can be fitted using the geometrical scaling parameters \( H, L \) and \( n \) to the observed terrain profile curvature, given a known soil depth function and a proper choice of \( \omega \) to represent the observed hillslope width function (Troch et al., 2002).

![Figure 1](image-url). (a) A three-dimensional view of a convergent hillslope overlying a straight bedrock profile; (b) a definition sketch of the cross section of a one-dimensional hillslope aquifer overlying a bedrock with a constant bedrock slope angle (modified from Troch et al., 2003).
Hillslope hydrology

The hillslope hydrological model used here is the hillslope-storage Boussinesq (HSB) model for subsurface flow (Troch et al., 2003) on complex hillslopes. The Darcy equation along a unit-width hillslope with sloping bedrock reads

\[ q = -kh \left( \frac{\partial h}{\partial x} \cos \beta + \sin \beta \right) \]  

(2)

Substituting in the continuity equation...
Table II. Hydrological and geotechnical model parameters

<table>
<thead>
<tr>
<th>Parameter group</th>
<th>Parameter name</th>
<th>Symbol</th>
<th>Units</th>
<th>Value in this paper</th>
</tr>
</thead>
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<tr>
<td>Hydrological</td>
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</tr>
<tr>
<td></td>
<td>effective porosity</td>
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<td>–</td>
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<td></td>
<td>recharge</td>
<td>$N$</td>
<td>mm d$^{-1}$</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>Van Genuchten parameter</td>
<td>$\alpha_v$</td>
<td>m$^{-1}$</td>
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</tr>
<tr>
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<td>Van Genuchten parameter</td>
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<td></td>
<td>residual water content</td>
<td>$\theta_r$</td>
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<td></td>
<td>saturated water content</td>
<td>$\theta_s$</td>
<td>m$^3$ m$^{-3}$</td>
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</tr>
<tr>
<td>Geotechnical</td>
<td>effective soil cohesion</td>
<td>$c_e$</td>
<td>kN m$^{-2}$</td>
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</tr>
<tr>
<td></td>
<td>soil depth</td>
<td>$D$</td>
<td>m</td>
<td>2·0</td>
</tr>
<tr>
<td></td>
<td>effective angle of internal friction</td>
<td>$\phi$</td>
<td>°</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>slice length</td>
<td>$dx'$</td>
<td>m</td>
<td>0·5</td>
</tr>
<tr>
<td></td>
<td>saturated bulk specific weight</td>
<td>$\gamma_s$</td>
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</tr>
<tr>
<td></td>
<td>water specific weight</td>
<td>$\gamma_w$</td>
<td>kN m$^{-3}$</td>
<td>9·81</td>
</tr>
</tbody>
</table>

\[
j \frac{\partial h}{\partial t} = - \frac{\partial q}{\partial x} + N \tag{3}
\]

yields the Boussinesq equation (Boussinesq, 1877):

\[
\frac{\partial h}{\partial t} = \frac{k}{f} \left[ \frac{\partial}{\partial x} \left( h \frac{\partial h}{\partial x} \cos \beta + \frac{\partial h}{\partial x} \sin \beta \right) \right] + \frac{N}{f} \tag{4}
\]

where $h(x, t)$ is the elevation of the groundwater table measured perpendicular to the underlying impermeable layer which has a slope angle $\beta$, $k$ is the hydraulic conductivity, $f$ is the drainable porosity, $x$ is the distance from the outlet measured parallel to the impermeable layer, $t$ is time and $N$ represents the rainfall recharge to the groundwater table. Note that the flow lines for these equations are assumed to be parallel to the bedrock.

As the application of Equation (4) is limited to one-dimensional soil mantle, Troch et al. (2003) reformulated the continuity and Darcy equations in terms of storage along the hillslope, which leads to the hillslope storage Boussinesq (HSB) equation for subsurface flow on complex hillslopes:

\[
f \frac{\partial S}{\partial t} = k \cos \beta \frac{\partial}{\partial x} \left[ S \frac{\partial S}{\partial x} - S \frac{\partial w}{\partial x} \right] + k \sin \beta \frac{\partial S}{\partial x} + f N w \tag{5}
\]

where $S = S(x, t) = w h(x, t)$ is the subsurface water storage, $h = \bar{h}(x, t)$ is the water table height averaged over the width of the hillslope and $w(x)$ is the hillslope width function. The total storage capacity along the hillslope can be expressed as $S_c(x) = f w(x) \bar{D}(x)$, where $\bar{D}$ is the soil depth averaged over the width of the hillslope. Paniconi et al. (2003) have shown that the HSB model is able to capture the general features of the storage and outflow response of complex hillslopes, as compared with a 3D Richards equation based simulation.

Equation (5) allows us to investigate the hydrological behavior of the original Boussinesq equation on hillslopes of variable plan geometry. In order to analyze the effect of non-constant profile curvature, Hilberts et al. (2004) presented the HSB model as

\[
f \frac{\partial S}{\partial t} = k \cos \beta(x) \left[ \frac{\partial S}{\partial x} + S \frac{\partial B}{\partial x} + f S \frac{\partial \beta(x)}{\partial x} \right] + k \sin \beta(x) \left[ \frac{\partial S}{\partial x} - S \frac{\partial B}{\partial x} \right] + f N w \tag{6}
\]

where $B = \partial \bar{D}(x)/\partial x$. The model is solved numerically, can handle spatially and temporally variable parameters and allows for the computation of subsurface flow and saturation excess overland flow. By determining the saturated storage ($S$) at each time step (daily) of the simulation, the relative saturated storage ($\sigma = S/S_c$) is calculated and is directly coupled to the infinite slope stability method for determining the factor of safety.

During wet conditions, as will occur in the case of shallow landsliding, precipitation as an input parameter can be assumed equal to the recharge rate. The average soil moisture content in the unsaturated zone ($\theta$) can then be calculated according to Campbell (1974) by using Darcy’s law with the unit-gradient assumption as
where $\theta$ is the volumetric soil moisture content averaged over a depth $(D - h)$, $\lambda$ is the porosity, $N$ is the recharge rate, $k_s$ is the saturated hydraulic conductivity and $b$ is a pore size distribution parameter. Since $\lambda$ and $b$ are generally correlated with $k_s$, these were related to $k_s$ by linear regression with $\ln(k_s)$, fitted to the data provided by Clapp and Hornberger (1978). This yields $\lambda = -0.0147 \ln(k_s) + 0.545$ and $b = -1.24 \ln(k_s) + 15.3$, where $k_s$ is expressed in mm d$^{-1}$ (Teuling and Troch, 2005). Hence, at each time step (daily), the average soil moisture content in the unsaturated zone can be changed based on the recharge rate (rainfall).

**Hillslope stability**

Next, from the water table depth along the hillslope and the average soil moisture in the unsaturated zone, the factor of safety (FS) can be calculated at each time step. The safety factor is the ratio of the available shear strength to the minimum shear strength that is needed for equilibrium. Many variables are involved in slope stability evaluation and the calculation of FS requires geometrical data, physical data on the geologic materials and their shear-strength parameters (cohesion and angle of internal friction), information on pore-water pressures etc. In general, the infinite slope stability analysis has been widely applied in many investigations of natural slope stability (e.g. Montgomery and Dietrich, 1994; Wu and Sidle, 1995; Van Beek, 2002; Borgia et al., 2002; D’Odorico and Fagherazzi, 2003; Henrich and Crozier, 2004; Claessens, 2005; Rosso et al., 2006) because of its relative simplicity, particularly where the thickness of the soil mantle is much smaller than the length of the slope and where the failure plane is approximately parallel to the slope surface. The infinite slope model imposes the condition that the groundwater flow is parallel to the slope surface, which is consistent with the HSB model. In order to derive FS for the entire hillslope, Talebi et al. (2007) presented the shallow landslide safety factor for complex hillslopes by incorporating the relative saturated storage (the ratio between actual storage and storage capacity) in the safety factor formulation as follows:

$$\frac{1}{FS} = \frac{\int_0^L \left\{ c_i(x) + [(1 - \sigma(x))\gamma_m(x) + \sigma(x)\gamma_b(x)]D \cos \beta(x) \tan \phi \right\} dx}{\int_0^L \left\{ (1 - \sigma(x))\gamma_m(x) + \sigma(x)\gamma_b(x) \right\} D \sin \beta(x) dx}$$

(8)

where $L$ is the total length of hillslope (measured parallel to bedrock), $c_i$ is the total soil cohesion, $\phi$ is the angle of internal friction, $\beta$ is the slope angle and $\sigma(x)$ is the relative saturated storage. $\gamma_m$, $\gamma_s$ and $\gamma_b$ are respectively the moist, saturated and buoyant bulk density (note that in this equation $x$ is measured along the bedrock and $x = 0$ in the outlet).

With respect to the influence of soil suction on the slope stability, Fredlund et al. (1978) proposed a linear shear strength equation for unsaturated soils. According to this model, the total cohesion of the soil ($c_e$) can be calculated as

$$c_i = c_e + (u_a - u_e) \tan \phi$$

(9)

where $c_e$ is the effective cohesion of saturated soil (kPa) and $(u_a - u_e)$ is the matric suction of the soil on the plane of failure (kPa), where $u_a$ and $u_e$ are the pressures of pore air and pore water, respectively. For slope stability analysis, the pore air pressure is assumed to be atmospheric and constant. $\phi^b$ is the angle of shearing resistance with respect to matric suction. Vanapalli et al. (1996) proposed that the relation between $\phi^b$ and $\phi$ can be replaced by the degree of saturation as follows:

$$c_i = c_e + (u_a - u_e) \left( \frac{\theta - \theta_s}{\theta_t - \theta_s} \right) \tan \phi$$

(10)

Equation 10 shows how the total cohesion of the soil ($c_i$) is changed as a function of the soil moisture in the unsaturated zone ($\theta$) in each slice and each time step. As can be seen, by substituting the average soil moisture content and the average soil water suction into Equation (10), the soil cohesion in the unsaturated zone for each $x$ position along the hillslope is computed. Finally, the total soil cohesion at each $x$ position along the slip surface is calculated as a weighted average of the soil cohesion in the unsaturated and saturated zones.
Numerical analysis

After determining the plan shape and slope curvature (Equation (1)) for each hillslope (see Talebi et al., 2007), the dynamic model of hillslope stability is solved starting from the initial condition $h = 0$ at time $t = 0$ and the following steps are performed with a time step, $\Delta t$, of one day: (i) the saturated soil moisture storage and the relative saturated storage are calculated from Equation (6); (ii) the averaged soil moisture content in the unsaturated zone is estimated from Equation (7); (iii) the influence of soil suction on soil cohesion is computed from Equation (10); (iv) the factor of safety is determined from Equation (8) and (v) back to step (i) for the subsequent time step. For the saturated zone, it is assumed that the downhill boundary condition is $h(0, t) = 0$ and the uphill boundary condition is a zero-flux boundary, as are all sides and the bedrock. In this manner, $FS$ is obtained for the hillslope, with any temporal variations arising from dynamic hydrological responses. This will be used for the investigation of its relation to water table fluctuations and subsurface flow.

Results and Discussion

Effect of the unsaturated zone storage

In the HSB-SM, the rate of daily precipitation is substituted by the recharge rate directly. This means that the variations of the unsaturated zone storage have been calculated by Darcy’s law with a unit gradient assumption (see Equation (7)). To relax this assumption, the slope stability analysis has also been investigated based on coupling the saturated and unsaturated storages (see Hilberts et al., 2007). In this paper, to investigate the effect of the unsaturated zone storage on hillslope stability under time-varying conditions, the coupled model of saturated and unsaturated flow (Hilberts et al., 2007) has been combined with the infinite slope stability method. Figure 3 reports the values of the safety factor for hillslopes with different plan shapes using the original HSB model (solid line) and coupled HSB model (dashed line) (Hilberts et al., 2007). As can be seen (Figure 3), the two methods yield comparable results, illustrating that the hillslope stability is mainly determined by the water table dynamics (saturated soil moisture storage). Based on our analysis (see Figure 3), and for both methods, after the onset of rainfall, stability starts to decrease on all hillslopes. Obviously, after a certain period of rainfall (depending on hillslope type), stability becomes constant as the hydrological conditions approach the steady state. As can be seen, the $FS$ obtained from the original HSB model is a little less than that from the coupled HSB model; this is because the original HSB model produces a higher water table than the coupled HSB model (Hilberts et al., 2007).

These results are confirmed by other studies (e.g. Iverson, 2000; Iida, 2004; Rosso et al., 2006) that have shown that slope stability is controlled mainly by the water table dynamics. Therefore, with respect to the obtained results (Figure 3) and the limitation of the coupled model (Hilberts et al., 2007) for hillslopes with non-constant bedrock, we can safely use the original HSB model (Troch et al., 2003; Hilberts et al., 2004) for stability analysis on hillslopes with different plan shapes and profile curvatures. To compare the stabilities of these nine hillslopes by the Janbu method (Janbu, 1954), we also computed the $FS$ by two methods (Equation (8) and the Janbu method) (see Table III). As can be seen, the results of the two methods closely follow each other.

The relation between recharge rate and slope stability

The results of the stability analysis for three recharge rates equal to 10, 20 and 50 mm d$^{-1}$ (Figure 4) indicate clear differences in the stability of different hillslope types for the same soil conditions. In all cases (different recharge rates), convergent hillslopes with concave and straight profiles (Figure 4, nos 1 and 4) become unstable faster than others ($FS < 1$). This is because the convergent hillslopes drain much more slowly than the divergent hillslopes (Troch et al., 2003) and this process increases the saturated zone storage, which consequently decreases the factor of safety.

<table>
<thead>
<tr>
<th>Hillslope shapes</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>HSB-SM model</td>
<td>0.95</td>
<td>1.11</td>
<td>1.25</td>
<td>0.93</td>
<td>1.05</td>
<td>1.20</td>
<td>1.04</td>
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<tr>
<td>Janbu equation</td>
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<td>1.21</td>
<td>1.32</td>
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<td>1.13</td>
<td>1.27</td>
<td>1.02</td>
<td>1.07</td>
<td>1.29</td>
</tr>
</tbody>
</table>
quickly. In contrast, in the divergent hillslopes (Figure 4, nos 3, 6 and 9), which drain fast, even with 50 mm recharge per day, the slopes remain stable (FS > 1).

As can be seen, by increasing the recharge rate from 10 to 50 mm d\(^{-1}\) the stability duration decreases; however, the time to reach instability is different on all hillslopes. For instance, when the recharge is 10 and 20 mm d\(^{-1}\) all hillslopes are stable, whilst by increasing the recharge to 50 mm d\(^{-1}\) all convergent hillslopes become unstable after 8–10 d. As a result, it can be stated that from the plan shape viewpoint the convergent hillslopes, and from the profile curvature viewpoint the straight and concave hillslopes, become unstable faster than the others.
Figure 4. Variations of the safety factor in different hillslopes (Figure 2) by changing the recharge rate from 10 (dotted line) via 20 (dashed line) to 50 (solid line) mm d$^{-1}$ ($\beta = \phi$).

Effect of slope angle change

The hillslope stability has also been investigated for different slope angles. Figure 5 illustrates how, by varying the slope angle, FS is changed. The variations are completely regular on all hillslopes and for all slope angles. Comparison of Figures 4 and 5 shows that on a specific hillslope, by changing the slope angle FS also changes regularly, whilst by changing the plan shape and slope curvature the variations of FS are different for each hillslope type. This means that in addition to the slope angle the plan shape and slope curvature should be incorporated as key factors in slope stability models. As can be seen in Figure 5, when $\beta = \phi$, all convergent hillslopes (1, 4 and 7) become unstable (FS < 1) a few days after the onset of rainfall, whilst other hillslopes remain stable. This means that for steep slopes and for shallow landslides the effect of plan shape on hillslope stability is much more important than that of profile curvature.

The relation between subsurface flow and hillslope stability

With respect to the important role of subsurface flow on slope stability (see, e.g., Borga et al., 2002; Matsushi et al., 2006), the relation between variations of subsurface flow and FS has also been studied. Figure 6 illustrates how the variations of FS and subsurface flow at the outlet are almost each other’s mirror image: when subsurface flow increases, slope stability decreases on all hillslopes and vice versa. When the water table reaches a constant (steady state), subsurface flow and FS obviously also become constant. Also, at the moment when subsurface flow rate becomes a maximum, the FS approaches its minimum. Moreover, the time to reach the steady-state condition and constant FS are different on all hillslopes. On the concave convergent hillslopes (Figure 6, no. 1) it takes less time, and on the convex divergent hillslopes (Figure 6, no. 9) it takes longer than in the others. This is because the convergent hillslopes, due to the reduced flow domain near the outlet, drain much more slowly than the divergent hillslopes (Troch et al., 2003), and as a result build up saturated storage much more quickly. Note that when the subsurface flow reaches a constant (steady state) with respect to recharge rate ($N = 50$ mm d$^{-1}$) the subsurface flow at
Figure 5. Variations of the safety factor on different hillslopes (Figure 2) by changing the slope angle from 20 (dotted line) via 30 (dashed line) to 40 (solid line) degrees \((N = 50 \text{ mm d}^{-1})\).

The outlet for some hillslopes is less than 20 mm d\(^{-1}\). This is because on these hillslopes flow concentrates near the outlet region, resulting in overland flow. Finally, based on Figure 6 it can be concluded that when the hillslope shape changes from divergent to convergent (with the same rainfall) hillslopes become unstable earlier. On the other hand, when profile curvature changes from convex to concave, hillslopes also become unstable more quickly.

Sidle and Ochiai (2006) note that spatial and temporal variability in subsurface flow may be strongly linked to three-dimensional preferential flow networks at the hillslope scale and can exert a huge effect on landslide initiation. However, our model cannot address this issue, and to better understand the effect of preferential flow on hillslope stability it is necessary to evaluate or spatially simulate a likely array of pathways rather than simply preferential fluxes (Sidle and Ochiai, 2006).

**Conclusion**

In this paper we have presented a physically based hillslope stability model to investigate the hydrologic control of shallow landsliding for complex hillslopes (HSB-SM). The model is based on a combination of the hillslope-storage Boussinesq model (HSB) (Troch et al., 2003; Hilberts et al., 2004) and the infinite slope stability method based on the Mohr–Coulomb failure law. The HSB model is based on the continuity and Darcy equations in terms of storage along the hillslope. The resulting HSB-SM shows that the dynamic response of complex hillslopes during drainage and recharge events depends very much on the slope angle, plan shape and slope curvature. We have focused on the study of flow processes in the situation where the topographic relief and the shallow subsurface moisture control the storage and stability of the hillslope.

We have also combined the coupled system for soil moisture storage in the saturated and unsaturated zones (Hilberts et al., 2007) with the infinite slope method to investigate the effects of unsaturated storage variations on hillslope stability on complex hillslopes. Our analysis shows that there is not a large difference between the two methods, illustrating that the variations of the unsaturated zone storage can be ignored safely in the dynamic slope...
stability analysis for shallow landslides. Therefore, it can be concluded that the presented model (HSB-SM) can be used safely for hillslope stability analysis on complex hillslopes under dynamic hydrological conditions. Furthermore, this model allows us to investigate the relation between subsurface flow and slope stability on hillslopes with different plan shapes and different slope curvatures.

Based on our analysis, the minimum safety factor (FS) coincides with the maximum rate of subsurface flow. In fact, an increase of subsurface flow leads to a decrease of stability on all hillslopes and vice versa. Consequently, after a certain period of rainfall, the convergent hillslopes with concave and straight profiles become unstable more quickly than others. However, the divergent convex hillslopes remain stable even after intense rainfalls. Finally, it can be concluded that, in addition to the average bedrock slope angle, topographic characteristics (especially profile curvature and plan shape) of the hillslope control the subsurface flow, and this process affects hillslope stability by changing the soil strength. As the current model is limited to event-based analyses, further research is needed to present a probabilistic model of rainfall-triggered shallow landslides for complex hillslopes with changing rainfall input and soil depth.

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References

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